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ANALYSIS FINANCIAL AND ECONOMIC DATA USING JOINT TIME-FREQUENCY DISTRIBUTIONS

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Abstract: *Analysis of economic/financial time series in the frequency domain is a relative underexplored area of the literature, particularly when the statistical properties of a time series are time-varying (evolving). In this case, the spectral content of the series varies as time passes, making conventional Fourier theory inadequate to fully describe the cyclic characteristics of the series. Time-Frequency Conjuncture Representation techniques (TFR) overcome this problem to the best of their ability analyzing a given function of time (continuous or discrete) in the time domain and in frequency domain simultaneously.*

Keywords: *JTFA, discrete-time discrete-frequency, time series*

JEL Classification : *C23, C26, C38, C55, C81, C87*

1. Introduction

Time varying spectra is one of the most primitive sensations we experience since we are surrounded by light of changing color, by sounds of varying pitch and by many other phenomena whose spectral content vary with respect to time. With time-frequency signal analysis tools one can study and analyze these signals and identify the temporal localization of the signal's spectral components (Fig.1). In particular, the values of the time-frequency representation of the signal provide an indication of the specific times at which the spectral components of the signal are observed.

This is of special importance since the frequency contents of the majority signals encountered in our everyday life change over time, for example, biomedical signals, power signals, speech signals, stock indexes time series, and seismic signals. This fact is fostering the implementation of time frequency signal analysis tools in many important scientific and engineering applications, such power line signal analysis, SAR (Synthetic Aperture Radar), spread spectrum signal detection and the analysis of FM signals such as chirp signals (Fig. 3). This paper discusses the design, development and implementation of a discrete-time discrete-frequency (DTDF) environment for signal analysis using time-frequency representations. The DTDF environment makes a unified characterization of some well-known time-frequency signal analysis representations, such as the discrete ambiguity function (DAF), the Wigner Distribution (WD), the short-time Fourier transform (STFT) and the discrete wavelet transform (DWT). The DAF is a time-frequency representation that is broadly used in radar and sonar applications. The WD is widely used for signal detection and parameter estimation. The STFT is widely used in applications such as speech recognition and biological signals. A relatively new emerging tool is the discrete wavelet transform that is frequently used in applications such as transient signal analysis and image compression.

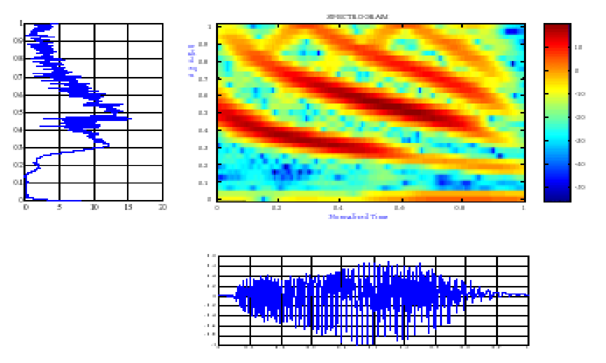


Fig. 1. Graphical representation of a time-frequency signal

This paper is organized as follows. Section 2 will delve into Time-Frequency Representation. Section 3 explains the mathematical representation of the DAF, the WD, the STFT and the DWT. In section 4 we present the computational environment. Section 5 then presents and discusses some results in applications such as STFT. Finally, some conclusions are discussed.

2. Time-Frequency Representation

The Fourier transform has been the most common tool to study a signal's frequency properties. It establishes in conjunction with the inverse Fourier transform a one-to-one relationship between the time domain and the frequency domain or spectrum space of the signal, which constitute two alternative ways of looking at a signal. Although the Fourier transform (Fig 4) allows a passage from one domain to the other, it does not allow for a simultaneous combination of the two domains.

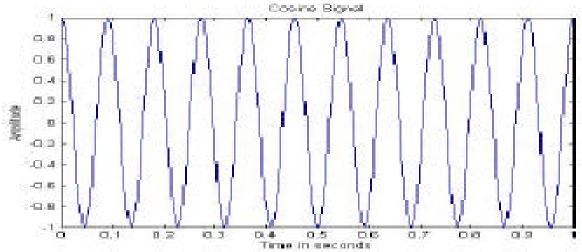


Fig. 2. Cosine: a single tone signal, its spectral characteristics does not vary with time

This presents a problem if we are interested in studying the frequency components of signals which are transient, or their spectral content vary as a function of time, e.g., economic data and speech signals.

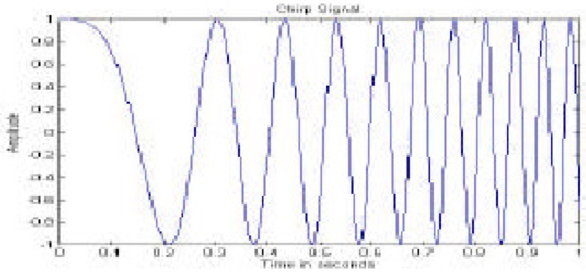


Fig. 3. Linear chirp signal (a band of frequencies), its spectral characteristics do vary with time.

Because of the need to explain such signals, the field of time-frequency signal analysis arose. Its main aim is to develop the physical and mathematical ideas needed to understand what a time-varying spectrum is and to use these methods for practical problems (Fig. 5). The tools already exist individually. It then emerges an imperative need of an environment that can create a uniform characterization of these tools for signal analysis in different engineering applications.

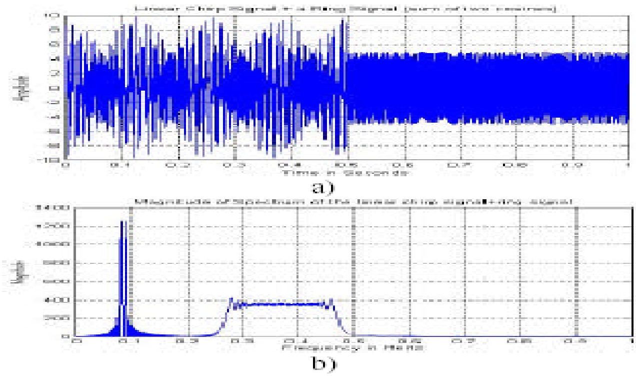


Fig. 4. A time-frequency signal: linear chirp signal + ring signal (sum of two cosines).a) Its graphical representation in time. b) Its Fourier transform

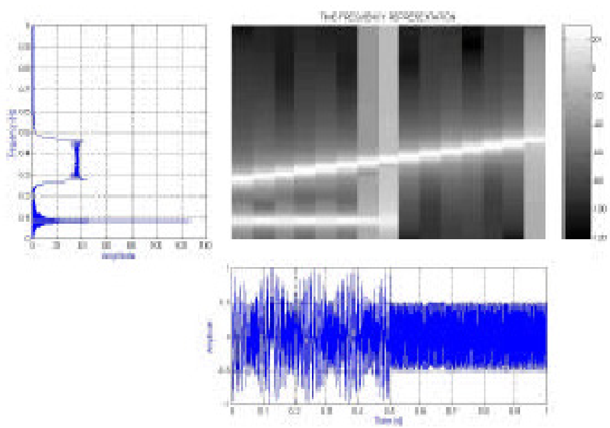


Fig. 5. Time-frequency (STFT) representation

Within time-frequency tools, the discrete ambiguity function, the short-time Fourier transform, the discrete Wigner distribution (Dumitescu et al., 2021) and the discrete wavelet transform seem to possess properties that can be very significant for their application to important problems encountered in time-frequency signal analysis. For this reason, we considered it a practical decision to concentrate in these four tools. We proceed to describe these tools in more detail (Dumitrescu, Minea, and Ciotarnae, 2019).

3. Ambiguity Function (DAF)

The DAF is a time-frequency representation that has as its objective to extract parameters such as frequency shift and time delay from a specific signal (parameter estimation), and is frequently used for signal estimation and Doppler effects. It is defined as follows:

$$\mathrm{A}_{x,y}[k,m]=\sum_{n=0}^{N-1}x[n]y^{*}[n+m]W_N^{kn} \tag{1}$$

where $W_N = e^{-j\frac{2\pi}{N}}, j=\sqrt{-1}$, x is the transmitted signal, k is the frequency shift, y is the received signal, and m is the time delay. Also, * denotes complex conjugation.

3.1.Discrete Wigner Distribution (WD)

The WD was first introduced in the field of physics. It is defined (Cohen, 1989; Oehlmann, and Brie, 2021) by:

$$W_{N,y}[m,k]=\frac{1}{2N}\sum_{n=0}^{N-1}x[n]y^{*}[m-n]W_{2N}^{k(2n-m)}, \tag{2}$$

where $W_N = e^{-j\frac{2\pi}{N}}, j=\sqrt{-1}$, x is the transmitted signal, k is the frequency shift, y is the received signal, and m is the time delay. Also, * denotes complex conjugation.

3.2. Short-Time Fourier Transform (STFT)

The STFT is a time-frequency tool that consists of a Fourier transform with a sliding time window. The time localization of frequency components is obtained by suitably pre-windowing the input signal. The STFT is:

$$S_x[n,k]=\sum_{m=0}^{M-1}x[m]w[m-n]W_M^{km} , \tag{3}$$

where $W_N = e^{-j\frac{2\pi}{N}}, j=\sqrt{-1}$, x is the input signal, w is the analysis window, k is the frequency offset, and m is the time delay.

3.3. Discrete Wavelet Transform (DWT)

It is defined (Cohen, 2022) as the sum over all the time of the signal multiplied by scaled, shifted versions of the wavelet function *g*. Given a finite energy signal *x(t)* and a normalized sampling period , $T_s = 1$ we can present a discrete wavelet analysis of the sampled sequence $x[n]=x(t)\big|_{t=nT_s}, n\in Z$, as follows:

$$c[s,b] = c[l,k] = \sum_{n=0}^{N-1} x[n] g_{l,k}[n] \ , \tag{4}$$

where $s = 2^l, b = k2^l, l, k \in Z$ and $g_{l,k}[n] = 2^{-\frac{l}{2}} g[2^{-l} n - k]$

The discrete synthesis operation can be presented by:

$$x(t) = \sum_{l \in Z} \sum_{n \in Z} c[l,k] \Psi_{l,k}(t) \ , \tag{5}$$

where $\Psi_{l,k}(t) = 2^{-\frac{l}{2}} \Psi(2^{-l} t - k), l, k \in Z$.

4. Computational environment

The main objective of the environment presented in this paper is the design, development and implementation of a single framework that combines discrete-time and discrete frequency concepts. The framework has been developed with the use of the software package LabView (Cohen, 1989) The environment implements discrete-time discrete-frequency versions of well-known time-frequency representations, such as the discrete ambiguity function (DAF), the discrete Wigner distribution (WD), the short-time Fourier transform (STFT), and the discrete wavelet transform (DWT).

The environment is divided in three major modules: Analysis and Synthesis module (Figure 6), Demonstration module and a Tutorial module. The Analysis and Synthesis Module provides the user with the essential tools for managing different types of signals as: *.JSON, *.DAT, *.CSV.

It possesses capabilities of retrieval and storage of data. In the environment, special attention was given to the graphical visualization and data rendering capabilities, since signal analysis is one of our man concerns. This part allows the user to choose the type of plot, the color-map, the shading and to rotate the figure. The environment also allows individual use of the tools and comparison of tool application results.

The discrete synthesis operation can be presented as follows: The Demonstration Module is a self-paced, step by step demonstration of the entire environment, including a user's guide, which has sufficient information on how to use the environment effectively. The tutorial module serves as a teaching and reference tool to the user for the technical aspects related with the development of the environment.

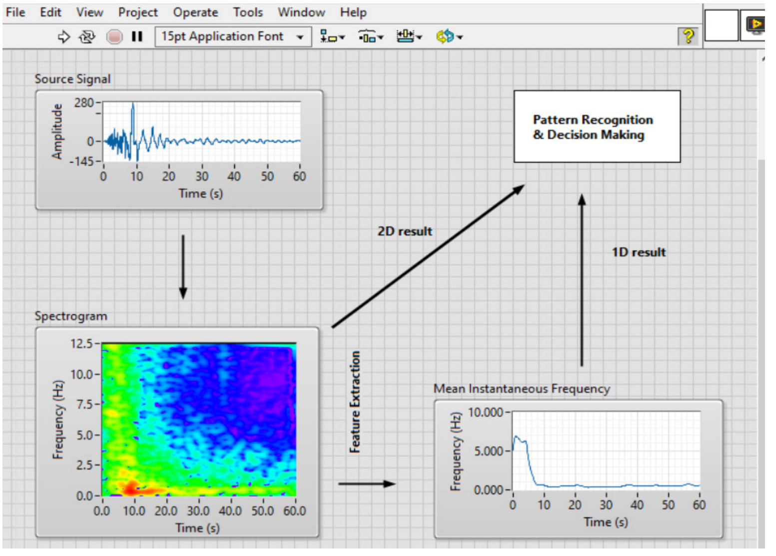


Fig.6. Computational Environment in LabView

In the following section some results obtained in applications such as financial and economic data (Dumitrescu, 2021; Nemtanu et al., 2015) signal analysis are discussed.

5. Application results. Design of Virtual Instrument

The data analysis VI contains three basic elements (Fig. 7). The raw data are retrieved from storage and any noise is removed (detrending). Then the traditional RMS (25ms windowed) of the signal from each brain zone and trial is computed, normalized, and made available for subsequent statistical comparison with the DTDF (JTFA Joint Time Frequency Analysis) data. Finally, the JTFA is performed, using the National Instruments JTFA Toolkit, and then the median frequency trace is calculated for each brain zone and trial. Figure 8 depicts the result of the STFT spectrogram from financial data (Dumitrescu, 2021; Mihura, 2021; Samat, and Mahesh, 2020; Olansen, 2002)

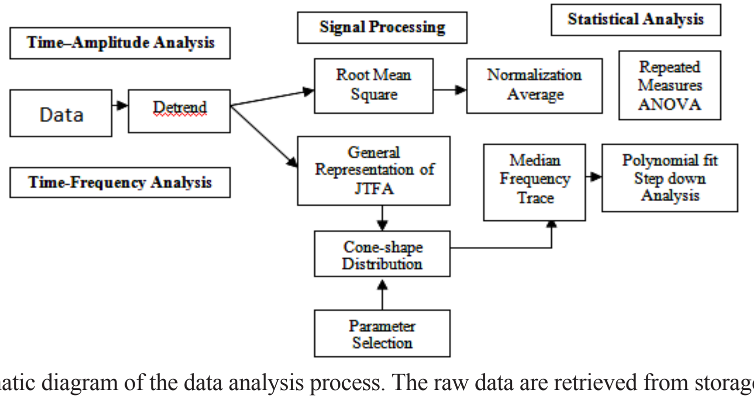
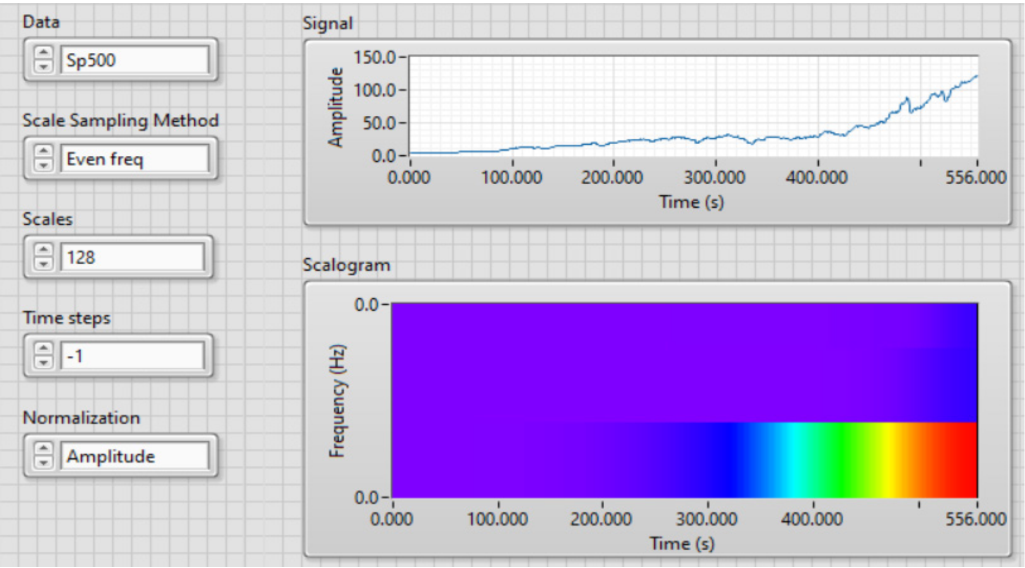
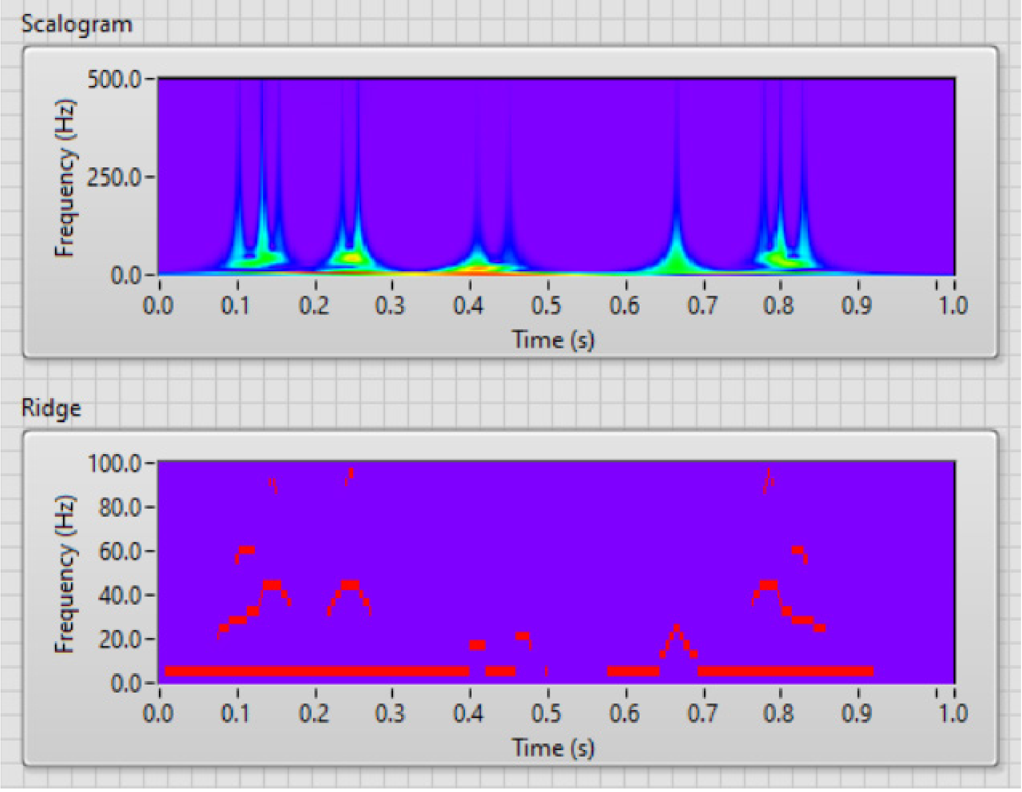


Fig.7. Schematic diagram of the data analysis process. The raw data are retrieved from storage, detrended, and then processed using traditional time-amplitude methods (top row) or using JTFA (bottom row).



(a)



(b)

Fig. 8. JTFA results from the financial series using (a) STFT analysis and (b) WT analysis

At the right of the spectrogram, the color bar helps determine the content of frequency at a specific time. The blue color (bottom of the color bar) means low quantity of frequency content and the red (top of the color bar) means high quantity of frequency content. From here we can determine that there is high content of frequency at low frequencies. Now the interference signal still appears, as a peak in the frequency domain and the extension of this maximum in time is also visible.

6. Conclusion

Like all other sciences, interest of economic has been the relationship between events and their causes. What causes market crisis? Are they caused by an external reason or an economic system's internal instability? If we consider the economic system as an organism rather, then business cycle can be thought of a heart rhythm or brain wave activity that provide some clues to diagnose the historical events.

A computational environment for the analysis of discrete time discrete frequency (DTDF) time-frequency signals has been presented. Application results using the DTDF environment were discussed, where the main objective was the detection of event-related (evoked) potentials result from external brain excitation that can degrade the regular brain activity can be improved using time-frequency signal analysis tools. For this the STFT was used. The event potentials can be detected and visualized easily in the time-frequency plane. This makes it easier to develop filters for the interference removal. Using JTFA, we hope to elucidate time-varying change in financial and economic data, revealing much more information about the time history of financial activity.

In this paper, we demonstrated that JTFA has great potential in studying stock market or financial date movements. By properly detrending, the complicated economic behavior could be characterized by a long-term trend plus short-time business cycle. Using the time-variant filter, we can remove the random noise in economic data financial changing. JTFA may be used for economic diagnosis of historical shocks and economic forecasting of evolution points.

References

Cohen, L., (1989). *Time frequency analysis*, proceeding of IEEE, vol 77, no. 7, July.

Cohen, K., (2022). *Wavelets-A new Orthonormal Basis*, Geophysics Departament, Center for Wave Phenomena, Colorado School of Mines.

Dumitescu, C., Costea, I., Cormos, A., and Semenescu, A., (2021). Automatic Detection of K-Complexes using Cohen Class Recursiveness & Reallocation Method and Deep Neural Network with EEG Signals, MDPI *Sensors* 2021

(IF 3.576 – 2021), *21*(21), 7230; https://doi.org/10.3390/s21217230, Special Issue: Advanced in Sleep Monitoring Sensors, Devices and Computational Technologies, WOS: 000719388200001.

Dumitrescu, C., Minea, M., and Ciotarnae, P., (2019). UAV Detection Employing Sensor Data Fusion and Artificial Intelligence, Information Systems Architecture and Technology. *Proceedings of 40th Anniversary International Conference on Information Systems Architecture and Technology – ISAT 2019.* Advances in Intelligent Systems and Computing, vol 1050. Springer, Cham, DOI: 10.1007/978-3-030-30440-9_13, Part of the Advances in Intelligent Systems and Computing book series (AISC, volume 1050), WOS:000564746100013.

Mihura, B., (2021). *LabVIEW For Data Acqusition.* Pretince Hall.

Nemtanu, F., Costea, I. M. and Dumitrescu C., (2015). *Spectral Analysis of Traffic Functions in Urban Areas*, PROMET - Traffic & Transportation, Vol. 27, No. 6, pp. 477-484 - WOS:000368321400003.

Oehlmann, H., and Brie, D., (2021). *Distribution de Wigner-Ville Locale pour la Reduction des Interferences*, Proc. Seizieme Colloque GRETSI, pg. 667 – 670.

Olansen, J., (2002). *Virtual Bio–Instrumentation Application LabVIEW*. Pretince Hall.

Samat, A., and Mahesh, L., (2020). *LabVIEW Signal Processing.* Pretince Hall.