METODS AND TECHNIQUES FOR ANALYZING THE POPULATION EVOLUTION

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Abstract
On our planet live 7 billion people, every five days another million are born. Currently, world population is 10 times higher than 400 years ago. The demographic explosion occurred due to improved living standards and health, which allowed people to live longer than in previous centuries.
Any economic process or phenomenon allows a mathematical description of its behaviour from an economic theory. It is very important to know the future evolution of the population and its structure therefore we using the forecasting models of the population. The paper presents the main models used in forecasting population and the advantages and disadvantages of each model.

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Introduction

In the process of modelling have been a number of approximations: approximation functional (approximating functions involved in functions as simple, such as linear) approximation distribution (approximate probability distributions of system components actual distributions classic often normal or exponential), approximate independence (real consideration of system components as independent random variables), approximate aggregation (more similar objects of one).

The study demographic phenomena and processes necessary knowledge is subordinated to the number and geographic distribution of the population, its structure by different features of the evolution of these structures as factors that determine the population status changes over time. Of the population growth patterns can include a range of models from the simple to the intricate. A review of the most significant is presented briefly below. The models are divided into two groups:

1. global forecasting models of population
2. predictive models of population structure

1. Global Forecasting Models of Population

Design models are models that global population projected total population without structures information. These are used to determine the total effective population perspective; we differentiate by gender, age or other characteristics.

Using this method is usually in estimating future population based on extrapolation. To do this, first determine the trend of population development over a certain period in the past by adjusting the data series extends this trend in the future for different horizons design.

a) Average annual growth rate method

$$S = \frac{P_1 - P_0}{n-1} \text{, where:}$$

$P_1 =$ population in the year of forecast

$P_0 =$ population a year before the calculation

n = number of years witch separated P_0 to P_1.

Number of population for each year of the forecast period is determined by adding the number of population since the calculation of average annual growth many times, how many years separated mementos future of computing time:

$$P_t = P_1 + n \cdot S \text{ , where:}$$

$P_t =$ population at time t witch to be projected

$P_1 =$ population in the year of forecast

$n =$ separating a number of years P_t to P_1.

The method can be used provided that, following the analysis of population trends from the previous period and studying the fate of the natural movement of population perspective it is concluded that the population will increase in arithmetic progression, the natural growth of its remaining constant for all period. Based on data on population trends in a previous period 5-10 years establishes annual average population growth.

This method applies to the design short-term as well as medium and long-term projections. Most effective in the short term but has since changes in the population is relatively constant over the short time period. Another drawback is the fact that this method does not present any information on the structure of the total number of the population or on the evolution of the aging population.

b) The average annual rate method

$$R = \sqrt{\frac{P_1}{P_0}} \text{ , where:}$$

$P_1 =$ population in the year of forecast

$P_0 =$ population a year before the calculation

$n =$ the number of years separating P_0 and P_1.

Population projection is calculated using the formula:

$P_t = P_1 \cdot R^n$ where:

$P_t =$ population at time t witch to be projected

$P_1 =$ population in the year of forecast

$n =$ number of years which separating P_t to P_1.

This method is relatively similar to that of before, the only difference being that if the average annual increase of the population requires a linear increase, the average annual growth is associated with an exponential equation.

This method is used with good results on relatively small period of time and requires a good knowledge of the earlier development of the population. As with the previous method that does not present any information on the structure of the total number of the population or on the evolution of the aging population.

c) Design methods population trend based on mathematical functions

If population linear function is determined by the relationship:

$P = a + b \cdot t$

If the exponential function:

$P = a \cdot b^t$,

If population parabolic function is determined by the relationship

$P = a \cdot t^2 + b \cdot t + c$, where the parameters a, b, c are determined by the least squares method.

From the methodological point of view is done in a similar manner to the case of using the two previous methods. It examines the graphic evolution of the population in the previous period and is determined by the trend that best matches the number of actual population likely to be linear function, exponential function, parabolic function, etc.

2. Predictive Models of Population Structure

Methods provide more information on the population structure and evolution of the degree of demographic aging.

a) Method of components

Population is determined by the relationship:

$$P_{t+1} = \sum_{15}^{49} F_x \cdot f_x + P_0 \cdot s_0 + P_1 \cdot s_1 + ... + P_{100} \cdot s_{100}, \text{ where:}$$

P_{t+1} = population in the year following the reference

F_x = age female population x (x ranging from 15 to 49 years)

f_x = fertility rate for age x

P_0 = population aged 0 years in the reference year

s_0 = survival rate (probability of reaching age population aged 0 to 1)

P_1 = population aged 1 year in reference

s_1 = survival rate (the probability that people age 1 to reach age 2)

....

P_{100} = population aged 100 years in the reference year

s_{100} = survival rate (probability-age population to reach age 101)

Components method is one method which gives a forecast of population by age. The method involves two stages: the first is to determine the number of survivors at each age, in other words the probability that the number of people shed x reaches an age $x+1$; and the second is to determine the number of născţi live in t.

To calculate the population P_{t+2} and so on, the same formula using age population projected for the previous year and considering that the survival and fertility rates remain constant over time. Fertility rates of mortality that are contingent on each individual age mortality tables that fertility.

b) Stochastic and deterministic models for forecasting population

$\min f(x)$

referring to:

$g(x) = 0$ $g(x) = 0$

$h(x) \geq 0$ (1,c)

$l \leq x \leq u$ l (1,d)

where $f : R^n \rightarrow R$ the objective function and restrictions model functions $g : R^n \rightarrow R^m$ and $h : R^n \rightarrow R^p$ is explained here as equalities and inequalities. *l* and *u* vectors defined restrictions on variables single borders. Such optimization model is called mathematical programming problem.

A dynamic optimization model in its general form, may be in the form of:

$$\min \varphi(x(t_f), t_f) + \int_{t_0}^{t_f} \Phi(x(t), u(t), t)dt$$

referring to:

$$g(x(t), u(t), t) = 0,$$
$$h(x(t), u(t), t) \geq 0,$$
$$a(t) \leq x(t) \leq b(t),$$
$$c(t) \leq u(t) \leq d(t),$$
$$t \in [t_0, t_f].$$

One issue that should be considered is linearity optimization models. Since optimization problems are very complex, involving thousands or tens of thousands of variables and constraints, mathematical problem solving these problems is extremely difficult. Therefore, modeliştii introduce some simplifications. One of the most common is the assumption that relations are linear optimization model. From the mathematical point of view, the linearity is very convenient. The most popular question of mathematical programming is linear programming, which has applications in virtually all areas of operation and known algorithms for solving polynomial. However, in reality, the linearity is very little present. This led to the development of nonlinear optimization methods, a very active area with remarkable results.

Often optimization models contain very complex algebraic expressions, which depend on a number of parameters, known as exogenous variables. Parameters values are specified and input into the model. The parameter values may be derived from other models or may be generated from the mental model of the modeler. The problem is very important to the accuracy of the parameters, their more accurate uncertainty has led to optimization of parameters, a stochastic optimization study dependence of parameters solutions, the introduction of the calculation interval, etc.

Finally, we note that often optimization models do not consider the so-called variable delay argument. Optimization models, especially dynamic optimization, containing variables are intractable and still do not have a theory stating optimality conditions. Delays in complex systems are important because they are a major source of instability.

Known many types of mathematical models and they can be classified in many ways. Models can be: static or dynamic, linear or nonlinear, deterministic or stochastic, etc. But one of the most profound mathematical models classification divides the optimizing and simulating. The distinction between the two classes of mathematical models is important because, on the one hand, fundamentally, they have different purposes and then, on the other hand, each based on other mathematical concepts.

Any optimization model has three components: the objective function, decision variables and constraints. The objective function defines the purpose or objective model. Obviously an optimization model can contain multiple objective functions. The objective function acts as a yardstick against which to do optimization. Decision variables are quantities that should we choose to optimize the objective function. We must make a distinction between the variables of the problem and its parameters. The model optimization parameters mean those sizes that define the model as an independent entity in the surrounding reality. Parameters have certain numerical values, well-defined and known. The decision variables are the unknown model. They can fit the model into various classes of models as values they can take. Such known models with varying real numbers, integers or Boolean variables are expressed as the corresponding crowds. Restrictions mathematical model is those relationships that constrain choice decision variables. Usually such an optimization model is given in the form of restrictions can be: static or dynamic, linear or nonlinear, deterministic or stochastic, etc. An optimization model considers three components in input and output generates the optimal decision variables, and the optimum value of the objective.

Most optimization models have limitations that must be known by those who use it. The first difficulty lies in the fact that an optimization model is, in a sense, rigid.

c) Model dynamic of population forecast

The dynamic model prediction of the population (IDMS) shows changes in the population as having a cumulative growth rate, after a number of intervals, equal to the product of long-term growth rate associated with each of these ranges.

This model focuses on the idea that the net birth rate is showing a sequence population projection matrix (Leslie). Leslie matrix elements of content they represent Net reproduction rate and the age at which women decide to have a child. In IDMS, the path of birth sequences generated by Leslie matrix can be analytically based.

A solution was offered explicit bowing at two rates in a population with two reproductive age. (Schoen and Kim 1994), but most cycles that characterize populations are too complex and are on a direct algebraic solution.

One of the models end to forecast population it is the Lotka model representing central model in math demographic. It assumes that a stable society while not lead to significant changes in terms of age-specific birth and mortaltăţii this leading to an exponential increase in birth and age structure unchanged (cf. Lotka 1939; Keyfitz 1968) .

1970 model is extended to the case multistadiu, that is considered more of a state and took into account the movements of these. (Land and Rogers 1982; Rogers 1975 Schoen 1988). They concluded the fixed rates can no longer be current. In our world characterized by rapid changes and fluctuations uncertain long-term, fixed rate option is unrealistic and uncertain term.

As it reached dinasmic need to develop a model that takes into account that vital rates (birth, death) change over time. In a pioneering work, Coal (1972) investigated the dynamic nature of rates, which affects them and found an approximate relationship between exchange rates and birth sequence is generated later. Lee (1974) considers dynamic in terms of population, external constraints. Kim (1987) makes a discrete analysis of the model and finds an algebraic solution linking them to the sequence of the rate of birth. However the solution was too complex to get its final shape. Cyclicality stable population studied and (Tuljapurkar 1990; Caswell 2001).

d) The Malthusian model

$Y = f(K,P)\cdot G$

$Y =$ total product

$P =$ total population

$K =$ total productive capital

$G =$ product multiplier which takes account of technical progress

Where f a homogeneous degree 1 and K and (in the absence of scale effect) and convex in K and P (relative increase f in K or P). It remains to define the evolution of K, P and G from one period to another.

For the capital, the variation is simply represented by the invested savings balance (with a saving rate constant) and the redemption of the:

$K" = uY - vK$

The population is expected to grow independently of the economic situation, with the rate determined as follows:

$$\frac{P'}{P} = r$$

In more simple models such as the Solow, is initially ignore technical progress ($G = 1$).The three foregoing equations

can be therefore calculating the logarithmic derivatives $\frac{K}{P}$:

$$\frac{\left(\frac{K}{P}\right)'}{\left(\frac{K}{P}\right)} = \frac{K'}{K} - \frac{P'}{P} = \frac{u \cdot f(K,P)}{K - v - r}$$

Since, by definition, $f(K,P)$ increase much less quickly than K, the second element is canceled for certain values of $\frac{K}{P}$.. In these conditions the derivative of $\frac{K}{P}$ becomes zero and we had a puncture stable equilibrium K which will increase at the same rate r as P well f. In the absence of technical progress, production is mainly responsible for the stagnation of the first people who would otherwise increase exponentially.

For the situation to improve obigatoriu need to register technical progress. In nine Solow's model, it is assumed that we have an exponential function of time rate s:

$$G = e \cdot st$$

and $Y = F(K,P)e \cdot st$

The model is based on two fundamental principles that seem to gideze after Malthus population trends: the first would be that food is essential to human existence and the second that fertility stems from the passion that ties the two sexes and remains now a necessity for perpetuating the species.

The two principles have been modeled in a mathematical expression that leads to the result that the population grows in geometric expression if it is not hampered by random factors.

On the other indicators subsistence increases only in arithmetical progression. As will be shown later first condition can turn into an increase in arithmetic when population exceeds subsistence level. The model assumes that a given country livelihoods are sufficient to meet the needs of citizens.

The Malthus saw along Timpu radical changes that led to a whole series of new models adapted to the new realities.

The total production tends to become proportional to strong population growth which is not the case in the Malthusian model for which production progress is endogenous. In order to reduce this issue has become customary to put a relationship between technical progress and the median income changes.

e) Models to analyze the impact of environmental change and globalization on demographic trends

The model contains several blocks, namely:

• *production unit*

Comprising three production sectors: industry intensive resources (producing final goods), consumer non-intensive resources (services) and mining (and intermediate goods).

• *investment block*

The share of total GDP of the country to investment depends positively on GDP per capita relative to GDP per capita initially in developed countries (as a measure of the need for minimum consumption) and negative for young people (aged between 0 -14 years) and the older of 65 years.

This link is one of the most important model and EXPIM the relationship:

$$c = 0.34 + (-0.07)\ln(PIB / PIB_0^R) + 0.7 \cdot pop(0-14) + 2.1 \cdot pop(65+)$$

where:

c is the consumption of GDP;

PIB is actually GDP / inhabitant;

PIB_0^R is GDP / capita initially in developed countries;

pop(0-14) represents the population with Varta 0-14 years;

pop(65+) represent the population with more than 65 years.

• *the quality of the environment block*

In this way consider air pollution and polluting revasărilor impact locally and regionally. The aim is to obtain information on how the emissions from economic activity and to a lesser extent the structure of the economy. The pollution resulting from energy use, which are assumed in the model, to be a linear function of resource intensive production in the industry as a logarithmic function depending on the final use of goods per capita.

• *population block*

The rate of infant mortality of children aged 1-5 years and elderly (age greater than 60 years) in the model are updated every 5 periods in accordance with changes in GDP / capita and time . Fertility is updated every five years in accordance with infant mortality (positively affected) and human capital (negative affect).

The model considers that the school population is aged 6-17 years targets and working population has aged 18-64 years.

Migrants are alleged to have aged 20-35 years, and the motivation for migration arising from their desire to maximize salary. The direction of migration is from low-wage countries to countries with high wages. These trajectories are studied using logit model.

In literature there are models that seek to determine the impact of globalization and climate change on demographic evoluţiiei. These models are used to quantify developments in socio-economic development of countries with different conditions in terms of endowment with natural resources, physical and human capital, technology and people, in a world with a special dynamic movement of goods persons and capital, with flexible structures of economies, etc.

The model seeks to quantify the impact of globalization in different countries, depending on specific conditions in the country. The model comprises the following major sections:

- global economic system;
- natural environmental resources system, on the quality of the environment, natural resources capacity to welfare, production and consumption in the economy;
- changes due to population growth and age distribution of it in each country considered (including international migration).

Consequently, the determinism and the frequency, time discrete model is followed by: i) steady, ii) optimization (maximize profit, maximize salary by international migration, final consumption of goods mixed joint investment, etc), iii) update (in production, number and age structure of the population, mortality rate and feritilitate, etc.). To these are added a set of information on international trade price indices.

Specialty literature includes an impressive number of different types of models which attempts, among other things, an assessment of the state where there is population in a certain area, determine the impact of various factors on indicators characteristics demographic phenomena, population prognosis term shield, medium or even long term. Using one or another of them depends on several factors: the database we have, time for the weather to be made, information should be highlighted etc.

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