ECONOMETRIC MODEL OF MULTIPLE EQUATION OF DIFFERENT SHAPE

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Summary:

Market economy has its own mechanisms which are subject to a continuous adjustment system through the action of specifics objective laws. In this context the role and the importance of knowing the manifestation way of demand and offer law have defining implications about the operational management of economical agents, about the commercial politic decisions, with spread effects on economic-financial performances. A useful methodological solution is offered by the econometric models based on multiple equations through which it can be estimated not only the physical equilibrium volume for demand and offer of merchandise but also the equilibrium price. The use of multiple equations models formed in more working variant, inevitably presents one useful informational value and required fundamental decision which can allow economical agent to work in ending conditions of profit activity.

Considering as support the graphical representation from Fig. 1, the study of demand and offer law will be approached also by a modeled variant in which will be used different shapes of regression equations, as follows:

Offer law = linear equation $\rightarrow \hat{y} = a + bx_1$

Demand law = hyperbolic equation $\rightarrow \hat{y}' = a' + b' \cdot 1/x_2$

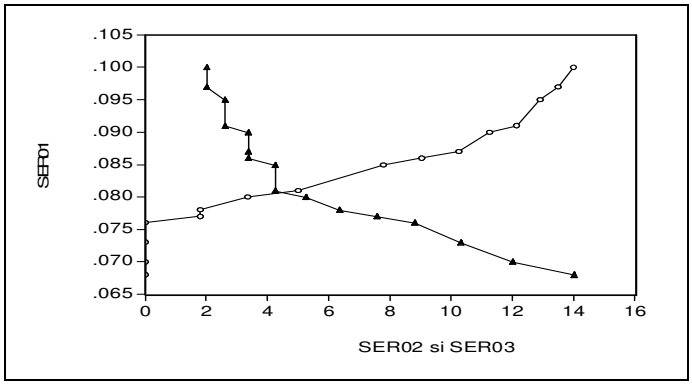

Fig. 1

Offer law

Table 1

Synoptic table results which attests the viability of parabolic regression equation as static modelled form of the correlation between price and physical volume of LCD monitors offer

Dependent variable: SER01 (Price) = y				
Method of smallest squares				
SER01 = 0,07243130411+0,001702470921*SER02 $\rightarrow \hat{y} = 0{,}072431 + 0{,}001702x_1$				
Observation number: 16				
Variables	Coefficient	Standard error of the coefficient	t-statistic	Probability (threshold signification)
SER02 (Offer) = x_1 b	0,001702	0,000117	14,48986	0,0000
C a	0,072431	0,000978	74,03482	0,0000
Determination coefficient R^2	0,937488	Dependent variable average		0,083375
Adjusted determination coefficient : R^2 adjusted (revised)	0,933023	Standard deviation of dependent variable		0,009612
Average (standard) error estimation of regression equation: $\pm\hat{\sigma}_{y.\hat{y}}$	0,002487	Statistical information criteria Akaike		-9,038618
Square residue sum	8,66E-05	Statistical criteria Schwarz		-8,942044
Log likelihood	74,30894	F-statistic		209,9561
Statistical coefficient Durbin-Watson	0,563898	Probability or threshold signification for F-statistic		0,000000

Table 2

Real level series, estimated level series of the dependent variable (price) dependent by the physical volume of LCD monitors offer and extension of residual term (linear regression)

n =16	Ser 01 Price (thousands EUR/piece) y	Estimated price values based on the regression equation $\hat{y}$	Residue values series $u = y - \hat{y}$	Extension residues $\pm\hat{\sigma}_{y.\hat{y}} = \pm 0{,}002487$ $-\hat{\sigma}_{y.\hat{y}}$ 0 $+\hat{\sigma}_{y.\hat{y}}$
1	0,06800	0,07243	-0,00443	|* , | , |
2	0,07000	0,07243	-0,00243	| * | , |
3	0,07300	0,07243	0,00057	| , |* , |
4	0,07600	0,07243	0,00357	| , | , * |
5	0,07700	0,07550	0,00150	| , | * , |
6	0,07800	0,07550	0,00250	| , | * |
7	0,08000	0,07813	0,00187	| , | *, |
8	0,08100	0,08094	5,6E-05	| , * , |

9	0,08500	0,08571	-0,00071	| , * | , |
10	0,08600	0,08784	-0,00184	| , * | , |
11	0,08700	0,08988	-0,00288	| *, | , |
12	0,09000	0,09158	-0,00158	| , * | , |
13	0,09100	0,09312	-0,00212	| ,* | , |
14	0,09500	0,09439	0,00061	| , |* , |
15	0,09700	0,09541	0,00159	| , | * , |
16	0,10000	0,09627	0,00373	| , | , * |
Total	1,33400	1,33400	0,00000	

Demand law

Table 3

Synoptic table results which attests the viability of parabolic regression equation as static modelled form of the correlation between price and physical volume of LCD monitors demand

Dependent variable: SER01 (Price) = y				
Method of smallest squares				
Observation number: 16				
SER01=C(1)+C(2) *1/SER03 $\rightarrow$ $\hat{y}' = a' + b' \cdot 1/x_2 \rightarrow \hat{y}' = 0{,}066743 + 0{,}066843 \cdot 1/x_2$				
	Coefficient	Standard error of the coefficient	*t*-statistic	Probability (threshold signification)
C(1) a'	0,066743	0,001097	60,86451	0,0000
C(2) b'	0,066843	0,003867	17,28733	0,0000
Determination coefficient R^2	0,955250	Dependent variable average		0,083375
Adjusted determination coefficient : R^2 adjusted (revised)	0,952054	Standard deviation of dependent variable		0,009612
Average (standard) error estimation of regression equation: $\pm\hat{\sigma}_{y.\hat{y}}$	0,002105	Statistical information criteria Akaike		-9,372897
Square residue sum	6,20E-05	Statistical criteria Schwarz		-9,276324
Log likelihood	76,98318	Statistical coefficient Durbin-Watson		2,105462

$$F-\text{statistic} = \frac{\Sigma(\hat{y}-\bar{y})^2}{k-1} : \frac{\Sigma(y-\hat{y})^2}{n-k} = \frac{0{,}001323858}{2-1} : \frac{0{,}0000620}{16-2} = \frac{0{,}001323858}{0{,}0000044286} = 298{,}9358$$

Table 4

Real level series, estimated level series of the dependent variable (price) dependent by the physical volume of LCD monitors offer and extension of residual term (hyperbolic regression)

n =16	Ser 01 Price (thousands EUR/piece) y	Estimated price values based on the regression equation $\hat{y}$	Residue values series $u = y - \hat{y}$	Extension residues $\pm\hat{\sigma}_{y.\hat{y}'} = \pm 0{,}002105$ $-\hat{\sigma}_{y.\hat{y}}$ 0 $+\hat{\sigma}_{y.\hat{y}}$
1	0,06800	0,07152	-0,00352	|* . | . |
2	0,07000	0,07231	-0,00231	| *. | . |
3	0,07300	0,07323	-0,00023	| . *| . |
4	0,07600	0,07434	0,00166	| . | *. |
5	0,07700	0,07560	0,00140	| . | *. |
6	0,07800	0,07727	0,00073	| . |* . |
7	0,08000	0,07947	0,00053	| . |* . |
8	0,08100	0,08247	-0,00147	| .* | . |
9	0,08500	0,08247	0,00253	| . | .* |
10	0,08600	0,08670	-0,00070	| . *| . |
11	0,08700	0,08670	0,00030	| . |* . |
12	0,09000	0,08670	0,00330	| . | . *|
13	0,09100	0,09245	-0,00145	| .* | . |
14	0,09500	0,09245	0,00255	| . | .* |
15	0,09700	0,10016	-0,00316	|* . | . |
16	0,10000	0,10016	-0,00016	| . *| . |
Total	1,33400	1,33400	0,00000	

Normality test of residual variables remedy

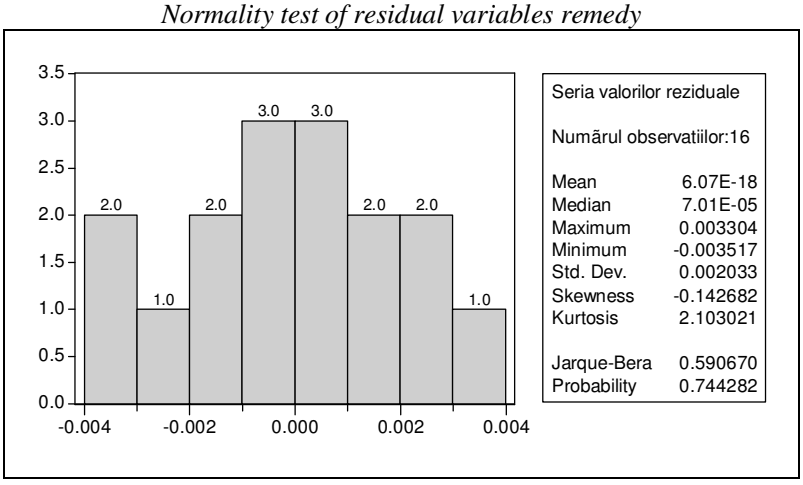

Table 5

The comparative situation of static indicators which attests the measure of econometric viability model between price and offer correlation and also between price and demand

Indicators	Offer law Linear model	Demand law Hyperbolic model
Correlation report: R	0,96824	0,977369
Determination coefficient: R^2	0,937488	0,955250
Statistical coefficient Durbin-Watson	0,563898	2,105462
Average (standard) error estimation of regression equation: - absolute expression: $\hat{\sigma}_{y,\hat{y}}$	0,002487	0,002105
Average (standard) error estimation of regression equation: - relative expression $\hat{V}_{y,\hat{y}} = (\hat{\sigma}_{y,\hat{y}} / \bar{y}) \cdot 100$	2,983%	2,525%
Statistical coefficient Jarque-Bera Probability associated to *J-B coefficient*	0,661530 0,718374	0,590670 0,744282
F-statistic	209,9561	298,9358
F-tabelar: $F_{P;\, f_1=k-1;\, f_2=n-k}$ P=95%; f_1=k-1=2-1=1; f_2=n-k=16-2=14	4,6	4,6
Probability (threshold signification) for F-statistic	0,000000	0,000000
Statistical information criteria Akaike	-9,038618	-9,372897
Statistical criteria Schwarz	-8,942044	-9,276324

In the context of 2nd way of calculation, analyse of the results from the comparative situation of the statistical indicators which attest the viability of the two regression equations, allows us to state the following conclusions:

- price and physical volume correlation of the offer and price and physical volume of the demand are expressed into linear regression equation for the offer law and into hyperbolic regression equation for demand law: $\hat{y} = 0{,}072431 + 0{,}001702 x_1$ şi $\hat{y}' = 0{,}066743 + 0{,}066843 \cdot 1/x_2$;

- the viability of the models is confirmed by the fact that the correlation report is statistical certified, been considerable different from 0, in the sense that "F Criteria" (Fisher distribution law, for a 95% probability and the number of freedom degrees f_1=1 and f_2=14). Between the variables of the two studied correlation systems it is a significant interdependence relationship, from statistical point of

view, and powerful as intensity, because the correlation report is almost equal to 1, $R_{yx_1} = 0{,}96824$ and $R_{yx_2} = 0{,}977369$;

- the relative expression of the standard estimation error of regression equation is equal to 2,983%, respectively 2,525%, is supplementing the informational support to conclude that the models are valid, because they are small values compared to the very restrictive pick of 5%;

- the residuals extension are is between the plus/minus limits 2,145 of the average estimation error of regression equation ($\pm 2{,}145 \cdot \hat{\sigma}_{y,\hat{y}}$), as per remedy law Student, for a signification pick of 5%, bilateral disposed, and n - k = 16 - 2 = 14 freedom degree, which confirm a correct econometric modulation of the studied correlation systems by regression equation: $\hat{y} = 0{,}072431 + 0{,}001702x_1$ and $\hat{y}' = 0{,}066743 + 0{,}066843 \cdot 1/x_2$;

- the parameters which localize the two regression equation present a statical segnification (are considerabe different zero) as per Student remedy law with bilateral disposal of segnification pick, confirming in this was the models viability;

- the error term values is distributed as per the normal repartition law with a probability of 71,84%, for offer regression equation and with a probability of 74,43%, for demand regression equation (certified by the statically criteria Jarque-Bera). Based on this probability which is higher that the minimum level of 60%, considered necessary for normality hypothesis acceptance, there is concluded that the econometric model with multiple equations is efficient from statistical point of view;

- The statistic coefficient Durbin-Watson present a value which is under the limit considered acceptable of 1,4 for the offer laniary model (0,563898) and a value of 2,105462, very close to the idea measure 2, for the demand hyperbolic model. In these conditions there is concluded that between the residual term values of offer linear model take place a relative auto-correction process which can negatively influence the quality of the estimations. This consideration is certified also by the disposal shape of error term values in the residual extension which does not present an obvious alternation. In this econometric study context it is mentioned that the observations number (n = 16) is relatively small and can be a cause of this phenomena;

The calculation of physical equilibrium model for demand and offer of LCD monitors and of equilibrium price, in the variant of different models representation, linear for offer law and hyperbolical for demand law, is as follows:

Offer law: $\hat{y} = 0{,}072431 + 0{,}001702 x_1$

Demand law: $\hat{y}' = 0{,}066743 + 0{,}066843 \cdot 1 / x_2$

$$x_1 = x_2 \;\rightarrow\; a + bx_1 = a' + b' / x_2 \;\rightarrow\; 0{,}072431 + 0{,}001702x = 0{,}066743 + 0{,}066843 \frac{1}{x}$$

$$\boxed{-0{,}066843 + 0{,}005688x + 0{,}001702x^2 = 0}$$

Results:

- the physical equilibrium volume for demand and offer of LCD monitors, for 2010 = 4,815 thousands piece.

$$x_1 = x_2 = \frac{-0{,}005688 \pm \sqrt{0{,}005688^2 - 4 \cdot (0{,}001702) \cdot (-0{,}066843)}}{2 \cdot 0{,}001702} =$$

$$= \frac{-0{,}005688 \pm \sqrt{0{,}000032353344 + 0{,}000455067744}}{0{,}003404} = \frac{-0{,}005688 \pm \sqrt{0{,}000487420088}}{0{,}003404} =$$

$$= \frac{-0{,}005688 \pm 0{,}022077615}{0{,}003404} = \text{două soluții:}$$

$$x^{01} = \frac{0{,}0163896015}{0{,}003404} = 4{,}815 \; thousands piece$$

$$x^{02} = \frac{-0{,}0277656015}{0{,}003404} = -8{,}157 \; thousands piece$$

Note: *The 2nd solution for x, x^{02}, it is not plausible and it is ignored*

- equilibrium price = 80,63 EUR/piece.

$\hat{y}_0 = a + bx = 0{,}072431 + 0{,}001702 \cdot 4{,}815 = 0{,}08062613$ mii euro/buc.; 80,63 euro/buc.

Table 6

The comparative situation of equilibrium values for two variants of econometric modulation with multiple equations of offer and demand of LCD monitors

Equilibrium values	Multiple equation model of parabolic type: - offer law = parabolic equation - demand law = parabolic equation	Multiple equation model: - offer law = linear equation - demand law = hyperbolic equation

- physical equilibrium volume for demand and offer of LCD monitor	5,532 thousands piece.	4,815 thousands piece.
- Equilibrium price	81.00 EUR/piece.	80.63 EUR/piece.

Considering some differences, mostly insignificant, from statistical point of view, the compared results exposed above allow us to consider that the best equilibrium values are related to the multiple equations model which uses the linear equation for offer and hyperbolic equation for demand.

Bibliography:

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