STOCHASTIC MODELS FOR CREDIT RISK

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Abstract

Risk is a fundamental factor of business because of any activity you can not get profit without risk. Therefore, any economic entity trying to maximize profits by managing risk specific field of activity and by avoiding or transferring risk that it does not want to take. It is evident that an efficient banking strategy should include both programs and bank risk management procedures designed to actually minimize the likelihood of such risks and potential exposure of the bank.

The paper presents some of the stochastic models used in the literature to determine and quantify the credit risk.

Keywords: credit risk, stochastic processes, exposure to the risk

JEL Clasiification: C19, C52, E51, E59

Introduction

Environment in which banks operate is changing, creating *new business opportunities*, but they involve also risks increasing and more complex. These, in turn, are a challenge, as real as it is threatening to traditional approaches of bank management.

Through the vital functions that are fulfilled, namely: the payment system, credit system and real economy, by vehicle of transmission of monetary policy national banking sector has a special importance for any national economy. Therefore, prudential regulation and supervision on the basis of the main components of the banking system is a premise and also an essential condition to achieve and maintain financial and economic health of a county.

The analysis of credit has both a quantitative dimension and a qualitative one. Quantitative dimension of credit analysis is based on specific activities of collecting, processing and interpretation of all client information, the bank has access.

Most times, it is considered that the main weaknesses in credit risk management are some administrative arrangements: poor selection of cases and improper internal supervision on the quality of borrowers. Risk of improper selection of cases can be minimized by: rigor in content records, internal quality assessment of clients on a unit by a scoring system, dual credit approval decision and establish a proper margin of perceived interest.

The results obtained in various fields by applying stochastic models are made in many financial and monetary issues such mathematical techniques to be used.

1. General characteristics of credit risk

Credit analysis represents the assessment of credit risk. Credit risk must be determined by what the bank expects to make from lending. The loan is bearing potential gains. Credit analysis is a process that must take place regularly: before granting credit to by credit of making decisions and then the loan maturity, the period of time, usually when the client's financial reports are made available.

Credit risk management objective is that, to allow a bank to achieve a sustainable risk profile by maintaining credit risk of exposures within acceptable limits, predefined in the risk strategy of the bank.

Credit risk has become a big issue in recent decades. Credit risk is defined as current or future risk of adversely affected profits and capital, for failure by the debtor of contractual obligations or damage situation.

If the bank lends a certain amount of money to a debtor, the debtor must repay the full amount, plus some compensation for loans received. Of course, there is always the danger that the debtor does not fulfill this obligation. Reasons for not paying credits can be different: a debtor's financial distress, fraud or something. In these cases the creditor loses some of his money, or all. How big the fraction of credit that the bank loses is, depends on various factors, including: the debtor's financial situation, bankruptcy costs, etc. Creditor must be aware of the risk. Therefore, he needs by the probability of credit losses for all portfolios of loans.

In the credit risk can appear a number of components, including: i) *counterparty risk* (the risk to record some losses on derivative contracts due to unable to pay of the counterparty), ii) *the risk of participation* (risk to register the loss due to changes in market value of shareholdings), iii) *country risk* (risk associated with credit risk, which is determined by economic, social and political conditions of the country of origin of the borrower), iv) *transfer risk* (occurs when the borrower's obligation is not expressed in its home country currency), v) *securitization risk* (resulting from securitization transactions in relation with the credit institution is), vi) *concentration risk*, vii) *residual risk* (determined by using the techniques to diminish credit risk).

Generally, in credit risk management can be applied to a number of general rules, including:

- approval of credit facilities, which shall be based on a risk assessment which takes into account both qualitative and quantitative elements;
- for framework contract, the bank is considering a number of issues: the specific nature of credit, contract terms borrowing, the profile of the exposure until maturity in terms of potential market developments, the existence of real or personal guarantees (if any), the probability of failure contractual obligations based on an internal rating system;
- credit decisions must be made by credit officers and credit committees, having competent to approval appropriate
- proposals and credit approvals should always be made in writing and retained;
- the credit facilities must be documented properly and in accordance with the terms and conditions of approval
- the credit risk exposures should be monitored, managed and reviewed periodically in relation to the limits established;
- the credit risk management, collaborate with business functions to manage loan portfolios and to implement measures to reduce credit risk for individual exposures or credit portfolios;
- there are processes for early identification of the assets with problems and for their corresponding classification.

Function of credit risk management, analyze loan portfolios and individual exposures and defines policies and processes for identifying, measuring, monitoring and controlling credit risk. Within that are contained a number of activities, including:

- definition of credit policy according to the strategy and risk profile of bank;

- transposition clearly and transparent the credit policy processes such that, to ensure the identification, measurement, monitoring and controlling of the credit risk;
- ensuring compliance with the policies on of the credit risk;
- reporting and monitoring credit risk exposures;
- development of quantitative analysis, on the credit portfolio and providing this analysis by the management of the bank.

1.1. Loans and credit risk in Romania

While maintaining some uncertainty regarding the evolution of Romanian economy, banks have adopted, further, a pro-cyclical prudent attitude in providing new loans, preferring to refinance the existing credit operations and purchase of government bonds.

In "Financial Stability Report-2011", elaborated by the National Bank of Romania, shows that "the future of non-performing loans and the challenges of the stock of loans granted to debtors not covered to currency risk remain important concerns on short-term of the central bank".

With the growth rate remained negative in 2010, the volume of bank assets and non-government credit registered a marginally growth in nominal terms. Thus, in 2010, the dynamic of net banking assets remained low (3.5% in nominal terms) than that recorded in 2009 (5%).

The central bank adopted in 2010 a series of measures to ensure the monetary conditions to encourage private sector demand for credit, however, banks have continued to show an aversion to risk more pronounced pro-cyclical. They have preferred to refinance the existing credit and investments in government securities. In this context, annual growth in nominal non-government credit was just 4.7% in 2010.

The evolution of non-government credit in 2010 and first half of 2011 was determined by the risk aversion persistence of banks, due to accumulation of large amounts of nonperforming loans and the negative impact on profitability indicators exercised by the provisioning costs.

In dynamic, the credit volume to the private sector decreased by about 3% from the second half of 2010, increased to 6% level since March 2011. This was due in large part, contraction of credit in national currency.

The volume of loans to households recorded a downward trend (-2%, in nominal terms in June 2011 compared with June 2010). In the case of the credit portfolio with counterpart companies has outlined as a positive development.

Even if the rate of credit risk was an upward trend in 2010, however, its growth was insignificant.

2. Stochastic models for credit risk assessment - theoretical formulation

Analysis and forecast of the credit risk was an intense preoccupation for specialists in the field since the seventh decade of last century. In this context may be mentioned some of the reference works in the field: Dellacherie (1970, 1972), Chou and Meyer (1975), Dellacherie and Meyer (1978a, 1978b), Davis (1976), Elliott (1977), Jeulin and Yor (1978), Mazziotto and Szpirglas (1979), Jeulin (1980), Brémaud (1981), Artzner and Delbaen (1995), Duffie et al. (1996), Duffie (1998b), Lando (1998), Kusuoka (1999), Elliott et al. (2000), Bélanger et al. (2001), Israel et al. (2001), Jeanblanc and Rutkowski (2000a, 2000b, 2001).

Also, have to be mentioned the approaches in reduced form, commonly referred to as intensity-based approach. This approach was initiated by Pye (1974) and Litterman and Iben (1991) and then formalized independently by Lando (1994), Jarrow and Turnbull (1995), and Madan and Unal (1998). But elements of this approach appear to Hull and White (1995), Das and Tufano (1996), Duff et al. (1996), Schönbucher (1996), Lando (1997, 1998), Monkkonen (1997), Lotz (1998, 1999), and Collin-Dufresne and Solnik (2001).

Assuming that the bank has a full portfolio of loans, denoted $PF = \{1, \ldots, N\}$. A debtor may have one or more loans. The credit loss L^i on loan $i \in PF$ can be written with the relationship:

$$L^i = EAD_i \cdot LGD_i \cdot 1_{\{\{\tau_i \le T_i\}\}} \quad (1)$$

where: EAD is exposure at default;

LGD =1- recovery rate (RR)

τ_i is the default time of the debtor (possibly infinite if the debtor will not default);

T_i is the maturity of the loan *i*;

The credit portfolio loss L is also called the aggregate loss. A portfolio credit risk model is interested in the estimation of the probability distribution of L and the time evolution of the loss. Let us introduce the loss process of loan *i*:

$$L_i^t = EAD_i \cdot LGD_i \cdot 1_{\{\tau_i \le \min\{t, T_i\}\}} \quad (2)$$

Then the loss process of the whole portfolio is

$$L_t = \sum_{i \in PF} EAD_i \cdot LGD_i \cdot 1_{\{\{\tau_i \le T_i\}\}} \quad (3)$$

with the loss process L_t is a non-decreasing stochastic process.

The bank managers are often interested in other statistics then the expected and unexpected loss. The main example is the credit (*VaR*) - value at risk which is a loss boundary which will not be crossed with a very high probability, typically 99%. More rigorously the credit VaR_t^α is defined as

$$VaR_t^\alpha = \inf\{l \in R | P[L_t > 1] < 1 - \alpha\} \quad (4)$$

The *VaR* is a very common used risk measure in practice, but it has a big disadvantage since it does not tell anything about the height of the loss if the loss is higher then *VaR*. It can be fixed by introducing the risk measure *CVaR - conditional value at risk* that is also often called the *expected shortfall* and is given by

$$CVaR_t^\alpha = E\lfloor L_t | L_t > VaR_t^\alpha \rfloor \quad (5)$$

There are many ways to model the process L_t. One can try to model it as an aggregate process which we will refer to as aggregate models. One way is to model the movement of every stock and the correlations between them. Another way is to model the index as some stochastic process. In that case some information is lost, but the modeling is easier. The aggregate loss process can be modeled using many approaches: time series, non-decreasing Markov chain, (Lévy) subordinator, or Stochastic Time Componentwise Optimization (STCO) method.

2.1. Stochastic Time Componentwise Optimization method (SCTO)

Model assumptions:

- data come from discrete annul observations, for a period of *m* years;
- the system operates according to some stochastic transformations in time, which are independent of a Markov chain;
- let Q be the valid generator matrix and $T_1, T_2, ..., T_m$ be some nonnegative random variables, and $t_1, t_2, ..., t_m$ be their realizations;
- in the i-th year, the system follows the continuous-time Markov chain, with the generator $t_i Q$. Therefore be estimated the variables $\hat{t}_1, \hat{t}_2, ..., \hat{t}_m$ and matrix $\hat{Q}$, conditioned on partially observed data;

- non-uniqueness of variables $\hat{t}_1, \hat{t}_2, ..., \hat{t}_m$ and matrix $\hat{Q}$, requires adding an additional condition on variables $\hat{t}_1, \hat{t}_2, ..., \hat{t}_m$, for example: $\sum_{i=1}^{m} \hat{t}_i = m$;

The aim of this method is to minimize credit risk, namely

$$\min\left(\sum_{i=1}^{m} \left\| \exp(t_i\, Q) - \hat{P}_i \right\|\right) \tag{6}$$

with restrictions:

$$\begin{cases} q_{ii} \le 0 & i = \overline{1,m} \\ q_{ij} \ge 0 & i \ne j \end{cases} \tag{7}$$

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$$\begin{cases} \sum_{k=1}^{N} q_{ij} = 0 \\ \sum_{i=1}^{m} t_i = m \quad ; \quad t_i \ge 0 \end{cases} \tag{8}$$

where || · || is the Euclidean norm

Briefly, the stages of STCO method are:

1) Compute Q as an average generator as described above*:*

$$\overline{\hat{Q}} = \frac{1}{m} \sum_{i=1}^{m} \hat{Q}_i \tag{9}$$

2) are determined the estimators for variables $t_i > 0$:

$$\hat{t}_i = \arg\min_{t \ge 0} \left\| \exp\left(t\hat{Q}\right) - \hat{P}_i \right\| \tag{10}$$

Since $\left\| \exp\left(t\hat{Q}\right) - \hat{P}_i \right\|$ is convex in *t* and for $t \to \infty$, atunci, there exists the unique minimum which can be easily found. Then it is necessary to rescale vector *t,* to hold $\sum_{i=1}^{m} t_i = m$ and calculate the parameters:

$$D = \sum_{i=1}^{m} \hat{t}_i \tag{11}$$

$$t = \hat{t}\, \frac{m}{D} \tag{12}$$

$$Q = \hat{Q}\, \frac{D}{m} \tag{13}$$

Therefore, this simple approach divides the optimization problem in two steps:

i) determining the estimator of the generator matrix, ii) estimate of the values t_i .

3) If $i \neq j$, with $i = \overline{1, N}$; $j = \overline{1, N}$; Q matrix elements are calculated using the formula:

$$q_{ij} = \arg \min_{q_{ij} \in [0,c]} \sum_{k=1}^{m} \left\| \exp\left(t_k \hat{Q}(q_{ij})\right) - \hat{P}_k \right\| \tag{14}$$

where *c* is chosen in such way that it is safely higher than any intensity and also higher then possible values of t_i (most times, it is considered c = 100).

4) Return *Q* as the optimal solution.

2.1.1. Estimations with STCO method

In the proposed analysis has found that the number of companies analyzed is n_{ij}^k , which started the *k*-th year in rating *i* and ended in rating *j*, and $n_{i\cdot}^k$ is the number of companies started in rating *i.*

Therefore, P_k matrix elements are given by:

$$p_{ij}^k = \frac{n_{ij}^k}{n_{i\cdot}^k} \tag{15}$$

Companies considered for this analysis were attributed to one rating: {*AAA, AA, A, BBB, BB, B, CCC, D}*. The best rating is AAA and D is a default.

Applying the STCO method to estimate the generator matrix *Q* and the variables T_k , has led to obtaining the data from Table 1.

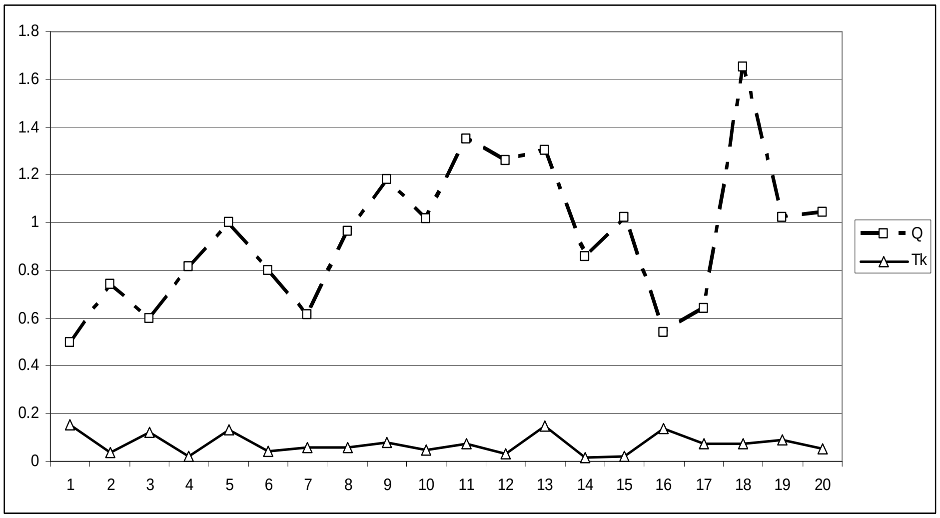
Tabeul nr.1 ***Multi–year generator*** Q^{STCO} ***with changed time***

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	-0.0989	0.0815	0.0018	0.0006	0.0005	0	0	0
AA	0.0052	-0.0869	0.0845	0.0029	0.0006	0.0006	0.0002	0
A	0.0002	0.0049	-0.0857	0.0641	0.0038	0.0032	0.0002	0.0003
BBB	0.0002	0.0017	0.0483	-0.1210	0.0552	0.0070	0.0017	0.0023
BB	0.0003	0.0008	0.0009	0.0765	-0.2125	0.1020	0.0151	0.0092
B	0	0.0010	0.0022	0.0010	0.0683	-0.2430	0.0599	0.0771
CCC	0	0	0.0032	0.0088	0.0136	0.1883	-0.6641	0.4122
D	0	0	0	0	0	0	0	0

Source: authors' calculations

Using STCO method, were estimated the Euclidean distances (P_k) and their evolution over time, putting into evidence the time period where they were significantly higher (Figure 1).

Figura 1 ***Estimates of the components of transition matrix, through STCO method***



Source: authors' calculations

The method allows the determination of loan repayment matrix elements STCO for each economic entity analyzed. However, this method allows the realization of correlations between economic developments and credit risk of banking entities.

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