OPTIMIZING THE PORTFOLIO OF ASSETS, ACCORDING TO THE MARKOWITZ MODEL

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Abstract:

One can argue – with a degree of certainty – that finance experts had long realised that the decision to invest needed to take into account both the profitability and the risk associated with the assets, but that used to be done mostly at an empirical level. The major contribution of professor H. Markowitz was that he was the first to put forward a concrete model for optimizing the selection of assets for investment portfolios under uncertain circumstances. More specifically, Markowitz showed how efficient portfolios (those that maximize expected profitability at a given risk level) can be put together, even when they consist only of riscky assets. The simplicity and the elegance of his result, as well as the high degree of practical applicability made it a very popular model, a true landmark in modern finances. In fact, H. Markowitz was awarded the Nobel Prize for economy in 1990, along with M. Miller and W. Sharpe.

Key words: Efficient portfolio, profitability, risk, volatility, optimum, variance

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Within this article we will outline the practical use of risk and portfolio management models. The example will refer to a number of 15 shares listed on stock markets in Germany (BMW – BMW, Commerzbank – CMB, Daimler – DAI, METRO – MET, Siemens – SIE), Poland (CEZ – CEZ, Global Trade Center – GTC, Lotos Group – LTS, PKO Bank Polski – PKO, TVN – TVN) and Romania (BRD – BRD, Oltchim – OLT, Rompetrol – RRC, SIF 5 – SIF5, Turbomecaninca – TBM). We analyzed their daily price between 01.01.2009 and 01.05.2011.

We will start by presenting the Markowitz model for selecting the assets, based on an analysis which takes into account the expected profitability and the assumed risk, as investors do not intend to put their capital into risk-free assets (investors' portfolios consist of risky assets only).

We shall use the simulation technique to generate a large amount of possible values for the yield-risk ratio, for portfolios consisting of the above stocks, thus defining the Markowitz efficient frontier. Note that we are analyzing a situation where the market permits short selling.

First, we aim to demonstrate how the Markowitz portfolio optimization model can be used for asset selection and portfolio management under the circumstances provided by integrated capital markets.

The daily average yields and average volatility during the considered period of time for the 15 listed companies are shown in the following table:

Company	**Daily yield**	**Daily volatility**
BMW	0.2475%	2.4665%
CMB	0.1697%	3.4304%
DAI	0.1940%	2.6182%
MET	0.1696%	2.0703%
SIE	0.1643%	2.0801%
CEZ	0.0217%	1.8025%
GTC	0.0905%	2.5091%
LTS	0.2620%	2.6726%
PKO	0.0978%	2.4725%
TVN	0.0996%	2.7610%
BRD	0.1326%	2.8527%
OLT	0.1873%	4.1731%
RRC	0.2242%	3.1411%
SIF5	0.2088%	3.4181%
TBM	0.1154%	3.9104%

Table 1. ***Dailiy yield and volatility levels***

(for the period between 01.01.2009 and 01.05.2011) for the 15 stocks considered in the analysis.

Source: own calculations.

The covariance matrix and the correlation matrix for the 15 stocks are presented as follows:

Ω	BMW	CMB	DAI	MET	SIE	CEZ	GTC	LTS	PKO	TVN	BRD	OLT	RRC	SIF5	TBM
BMW	0.000608	0.000369	0.000521	0.000228	0.000157	(0.000005)	(0.000027)	0.000044	0.000037	0.000070	0.000051	(0.000025)	(0.000056)	0.000012	(0.000076)
CMB	0.000369	0.001177	0.000436	0.000266	0.000152	(0.000029)	(0.000077)	(0.000086)	(0.000038)	0.000004	0.000020	0.000082	0.000056	0.000023	(0.000029)
DAI	0.000521	0.000436	0.000685	0.000262	0.000181	(0.000008)	(0.000048)	0.000038	0.000039	0.000065	0.000089	0.000068	(0.000032)	0.000039	(0.000009)
MET	0.000228	0.000266	0.000262	0.000429	0.000120	(0.000001)	(0.000016)	0.000007	0.000023	0.000026	0.000022	0.000009	(0.000020)	0.000029	(0.000021)
SIE	0.000157	0.000152	0.000181	0.000120	0.000433	0.000004	(0.000012)	0.000008	0.000024	0.000034	(0.000032)	0.000008	(0.000016)	(0.000014)	0.000004
CEZ	(0.000005)	(0.000029)	(0.000008)	(0.000001)	0.000004	0.000325	0.000131	0.000103	0.000077	0.000115	0.000038	(0.000001)	0.000021	0.000065	0.000016
GTC	(0.000027)	(0.000077)	(0.000048)	(0.000016)	(0.000012)	0.000131	0.000630	0.000308	0.000274	0.000359	(0.000023)	0.000059	0.000034	0.000059	0.000050
LTS	0.000044	(0.000086)	0.000038	0.000007	0.000008	0.000103	0.000308	0.000714	0.000344	0.000362	0.000057	0.000116	0.000016	0.000138	(0.000032)
PKO	0.000037	(0.000038)	0.000039	0.000023	0.000024	0.000077	0.000274	0.000344	0.000611	0.000341	0.000047	0.000069	(0.000029)	0.000092	(0.000006)
TVN	0.000070	0.000004	0.000065	0.000026	0.000034	0.000115	0.000359	0.000362	0.000341	0.000762	(0.000015)	0.000083	0.000055	0.000076	(0.000015)
BRD	0.000051	0.000020	0.000089	0.000022	(0.000032)	0.000038	(0.000023)	0.000057	0.000047	(0.000015)	0.000814	0.000099	(0.000038)	0.000275	0.000098
OLT	(0.000025)	0.000082	0.000068	0.000009	0.000008	(0.000001)	0.000059	0.000116	0.000069	0.000083	0.000099	0.001741	0.000080	0.000084	(0.000040)
RRC	(0.000056)	0.000056	(0.000032)	(0.000020)	(0.000016)	0.000021	0.000034	0.000016	(0.000029)	0.000055	(0.000038)	0.000080	0.000987	(0.000050)	(0.000019)
SIF5	0.000012	0.000023	0.000039	0.000029	(0.000014)	0.000065	0.000059	0.000138	0.000092	0.000076	0.000275	0.000084	(0.000050)	0.001168	0.000047
TBM	(0.000076)	(0.000029)	(0.000009)	(0.000021)	0.000004	0.000016	0.000050	(0.000032)	(0.000006)	(0.000015)	0.000098	(0.000040)	(0.000019)	0.000047	0.001529

Table 2. ***The covariance matrix (Ω) related to the yield of each considered stock.***
Source: own calculations

Naturally, elements on the main diagonal of the Ω matrix are the actual daily alternative yields provided by the stocks considered in our analysis.

	BMW	CMB	DAI	MET	SIE	CEZ	GTC	LTS	PKO	TVN	BRD	OLT	RRC	SIF5	TBM
BMW	1.0000	0.4356	0.8073	0.4472	0.3051	(0.0115)	(0.0431)	0.0674	0.0613	0.1034	0.0729	(0.0243)	(0.0727)	0.0143	(0.0786)
CMB	0.4356	1.0000	0.4857	0.3745	0.2137	(0.0468)	(0.0893)	(0.0941)	(0.0452)	0.0047	0.0202	0.0575	0.0521	0.0197	(0.0214)
DAI	0.8073	0.4857	1.0000	0.4839	0.3321	(0.0167)	(0.0728)	0.0537	0.0597	0.0904	0.1192	0.0626	(0.0386)	0.0435	(0.0092)
MET	0.4472	0.3745	0.4839	1.0000	0.2779	(0.0018)	(0.0311)	0.0123	0.0452	0.0452	0.0381	0.0104	(0.0306)	0.0409	(0.0253)
SIE	0.3051	0.2137	0.3321	0.2779	1.0000	0.0101	(0.0231)	0.0136	0.0461	0.0593	(0.0540)	0.0090	(0.0245)	(0.0201)	0.0050
CEZ	(0.0115)	(0.0468)	(0.0167)	(0.0018)	0.0101	1.0000	0.2899	0.2131	0.1727	0.2320	0.0732	(0.0016)	0.0363	0.1063	0.0226
GTC	(0.0431)	(0.0893)	(0.0728)	(0.0311)	(0.0231)	0.2899	1.0000	0.4587	0.4413	0.5179	(0.0316)	0.0566	0.0426	0.0687	0.0511
LTS	0.0674	(0.0941)	0.0537	0.0123	0.0136	0.2131	0.4587	1.0000	0.5202	0.4910	0.0754	0.1039	0.0193	0.1510	(0.0303)
PKO	0.0613	(0.0452)	0.0597	0.0452	0.0461	0.1727	0.4413	0.5202	1.0000	0.4994	0.0670	0.0670	(0.0371)	0.1083	(0.0065)
TVN	0.1034	0.0047	0.0904	0.0452	0.0593	0.2320	0.5179	0.4910	0.4994	1.0000	(0.0187)	0.0717	0.0630	0.0808	(0.0142)
BRD	0.0729	0.0202	0.1192	0.0381	(0.0540)	0.0732	(0.0316)	0.0754	0.0670	(0.0187)	1.0000	0.0832	(0.0420)	0.2824	0.0879
OLT	(0.0243)	0.0575	0.0626	0.0104	0.0090	(0.0016)	0.0566	0.1039	0.0670	0.0717	0.0832	1.0000	0.0607	0.0588	(0.0245)

RRC	(0.0727)	0.0521	(0.0386)	(0.0306)	(0.0245)	0.0363	0.0426	0.0193	(0.0371)	0.0630	(0.0420)	0.0607	1.0000	(0.0466)	(0.0157)
SIF5	0.0143	0.0197	0.0435	0.0409	(0.0201)	0.1063	0.0687	0.1510	0.1083	0.0808	0.2824	0.0588	(0.0466)	1.0000	0.0352
TBM	(0.0786)	(0.0214)	(0.0092)	(0.0253)	0.0050	0.0226	0.0511	(0.0303)	(0.0065)	(0.0142)	0.0879	(0.0245)	(0.0157)	0.0352	1.0000

Table 3. ***The correlation matrix related to the yield of each considered asset***
Source: own calculations.

As one can notice, correlations between the expected daily profitability of stocks are relatively weak. The only strong correlations (over 0.7) occur among shares of German auto giants, BMW and Daimler (which is explainable, given the industry in which the two companies operate). The fact that the profitability of different stocks is not strongly correlated means that benefits generated by the diversification of the portfolio will be significant, considerably reducing the risk associated with portfolios managed by investors.

Now we shall calculate the Markowitz efficient frontier, presuming that the market allows short selling. In the beginning, we aim to find out the coordinates of the R fundamental portfolios (the absolute minimum level portfolio) and U (maxim return portfolio if the short sales are not authorized on the market), using calculations $\begin{cases} R_R = \dfrac{B}{A} \\ \sigma_R = \sqrt{\dfrac{1}{A}} \\ x_R = \dfrac{1}{A}\Omega^{-1}U \end{cases}$ and $\begin{cases} R_U = \dfrac{C}{B} \\ \sigma_U = \dfrac{\sqrt{C}}{B} \\ x_U = \dfrac{1}{B}\Omega^{-1}R \end{cases}$. The coordinates (profitability expectations and volatility) of the two porfolios will be:

$$E(R_R) = 0.1399\%$$
$$\sigma_R = 0.9422\%$$

$$E(R_U) = 0.2314\%$$
$$\sigma_U = 1.2119\%,$$

and the structure vectors will be given by:

Action	x_R	x_U
BMW	11.45%	32.93%
CMB	2.02%	0.81%
DAI	-7.02%	-18.90%
MET	13.25%	14.15%
SIE	15.55%	17.43%
CEZ	20.61%	-4.63%
GTC	6.07%	2.04%
LTS	1.99%	23.79%

PKO	6.31%	-3.11%
TVN	-0.89%	-6.90%
BRD	7.97%	5.79%
OLT	3.84%	5.32%
RRC	9.68%	16.48%
SIF5	3.43%	8.21%
TBM	5.75%	6.58%

Table 4. ***Structure vectors of fundamental portfolios R and U***
Source: own calculations.

To find out the Markowits efficient frontier we shall simulate several values for the risc-profitability pair, using calculation formula $\sigma_P = \sqrt{\sigma_P^2} = \sqrt{\frac{1}{D}\left(AR^{*2} - 2BR^{*} + C\right)}$. The results of the simulation are as follows:

E(R$_P$)	**σ_P**
0.0000%	1.4980%
0.0100%	1.4343%
0.0200%	1.3726%
0.0300%	1.3133%
0.0400%	1.2567%
0.0500%	1.2032%
0.0600%	1.1533%
0.0700%	1.1074%
0.0800%	1.0660%
0.0900%	1.0297%
0.1000%	0.9990%
0.1100%	0.9745%
0.1200%	0.9566%
0.1300%	0.9458%
0.1400%	0.9422%
0.1500%	0.9459%
0.1600%	0.9569%
0.1700%	0.9749%
0.1800%	0.9996%
0.1900%	1.0304%
0.2000%	1.0668%
0.2100%	1.1083%

0.2200%	1.1543%
0.2300%	1.2044%
0.2400%	1.2579%
0.2500%	1.3145%
0.2600%	1.3739%
0.2700%	1.4356%
0.2800%	1.4994%
0.2900%	1.5651%
0.3000%	1.6323%
0.3100%	1.7010%
0.3200%	1.7709%
0.3300%	1.8419%

Table 5. ***Possible values of the expected profitability – assumed risk pair, presuming that the market allows short selling.***
Source: own calculations.

The graph on the following page illustrates the Markowits efficient frontier (the higher side, or that of dominating portfolios):

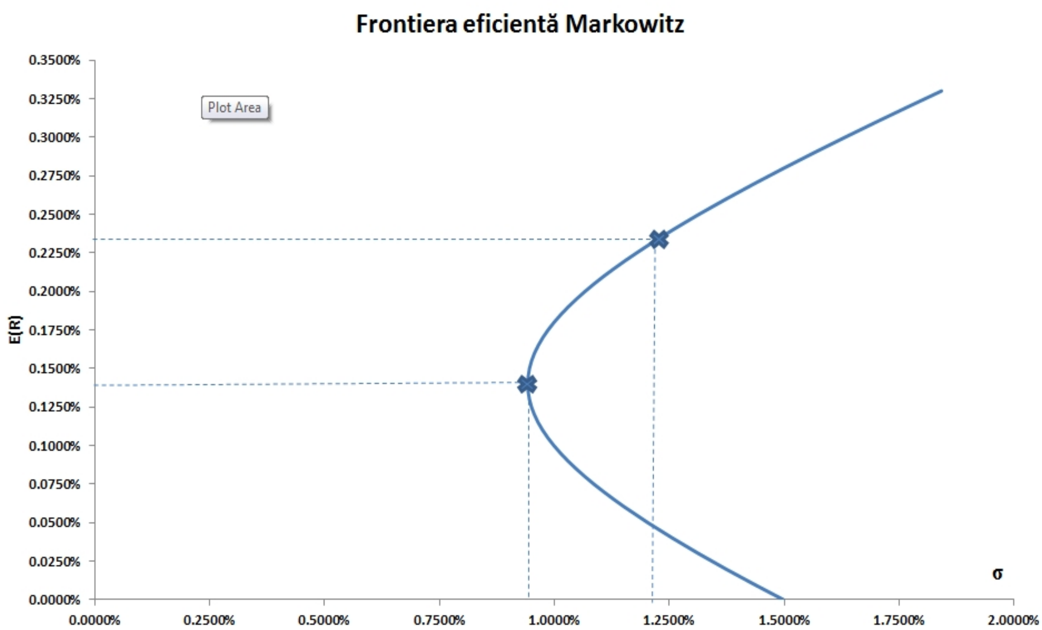

Image 1. ***The Markowitz efficient frontier when short selling is allowed***
Source: own calculations.

The lower side of the hyperbola (that of dominated portfolios) includes achievable but undesirable portfolios, given that portfolios on the Markowitz frontier assure a higher yield, at the same risk level.

Thus, by using the Markowitz model which scenario places us in a market where the risk-free assets are not quoted, the horizon of the portfolios made by the investors being limited to those 15 shares that were taken into consideration, we identified the Markowitz frontier (efficient set of portfolios). This represents the geometric locus of the portfolios that are characterized by minimal volatility at a certain level of the expected return. Hypothesis that we started from is that short sales are authorized on the market, which means that all these 15 shares could be used for the formation of efficient portfolios. In this case we have shown absolute minimum risk portfolios that investors can constitute and we graphically presented the Markowitz efficient frontier in the assumed risk plan - the expected return.

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