A FUZZY MODEL TO ESTIMATE ROMANIAN UNDERGROUND ECONOMY

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Abstract:

I propose here a model based on fuzzy logic in order to "quantify" Romanian underground economy. This approach starts from MIMIC model variables used by many authors in estimating underground economy in all over the world. I also assumed there can be establish a positive relation between a number of causal variables and underground economy.

The model uses a set of variables whose choice is based on both economic theory and empirical observations. These variables determinate underground activities. The choice of variables can be considered subjective and the input variables set can be modified depending on the availability of data need. However, it should be noted that each of these variables have a lesser or greater contribution to underground activities development.

Fuzzy logic language permits us to formulate rules such as "if the taxation rate is high, then the underground economy is large". Using statistical series it can be establish a basic "normal" value for a given period against which all the variables magnitude can be calculated. The "normal" value for each series and each year is actually an average of previous time values. Many rules can be formulated, but they depend on number and values of variables considered.

Keywords: fuzzy logic, linguistic variables, fuzzy model, underground economy, fiscal policy, underground economy fuzzy modelling, taxation rate, informal economic activities.

JEL Classification: C13, C61, D03, D84, E62, H30, O17.

1. Introduction

Taxation rate as a key parameter of fiscal policy is considered the main instrument of any economic stabilization and recovery program. Increasing tax rates may limit private investments and encourage business migration from formal to informal sector. The intensity of this transfer and evaluation of main parameters involved (economic and non-economic factors) represent the main purpose of underground economy modelling.

Underground economy estimation is difficult but not impossible. It is a real challenge for economic experts who concluded that any change in underground economy size is reflected by a change of main macroeconomic indicators. Based on these considerations there have been developed various models in order to facilitate the evaluation of causes and effects of informal economic activities and to estimate the size and dynamics of underground economy.

2. Preconditions of fuzzy logic application in economics

The universe is composed of several elements that can not be strictly defined or delimited. Starting from such observation, Lotfi Zadeh (considered the father of fuzzy logic) has decided to extend the two logical values defined by the pair {0, 1} to a continuous range [0, 1] by using a gradual transition from false to true[1].

Fuzzy sets are actually an extension of the mathematical set concept. A mathematical set is a collection of same feature objects. Fuzzy sets use more than one condition to establish a belonging criterion. Let's see the set of active persons from economy. A young man of 30 years old is certainly a part of this set, but an old person is excluded. What about persons of 40, 50, 60 or even 70 years old? Zadeh introduced the concept of membership level that allows a gradual transition from a set to another. The membership degree to a set is the main feature of a fuzzy set.

The membership degree is a precisely concept, but represents a subjective indicator depending on a certain context. Linguistic variables are another feature of fuzzy sets. Similar to arithmetical variables which are numerical values, fuzzy variables are specific values as words or sentences. The set of linguistic variables is generally named terms. Each member from a set of terms is a fuzzy variable defined on a basic variable.

Binary logic has emerged with language study. Sentences are true or false assertions, but not both simultaneously. However, a fuzzy logic sentence can be true or false, but it may have an intermediate value as "almost true", for example.

Fuzzy set theory and fuzzy functions are widely applied in computer science, systems analysis, electrical and electronic engineering and other complementary fields. Expert systems development supported fuzzy logic which has already taken place in our lives without even realize (automotive industry applications, domestic electronics etc.).

[1] Jantzen Jan – Tutorial on Fuzzy Logic, Technical University of Denmark, Department of Automation, Tech. Report no 98-E 868, 19 August 1998, pag. 2.

Although mostly used in sciences, the convenience of applying fuzzy logic in the social sciences has been limited for more or less psychological reasons. This is the case of economics. Timid attempts to use the theory of fuzzy sets in econometrics[1] occurred only after the mid-90s. This is the period when it has been tried to use fuzzy logic for non-linearity regression modelling and for investment behaviour prediction on a base of interest rate and transactions[2].

3. Methodological aspects

I've proposed to measure Romanian underground economy using a method not frequently exploited by economic problems analysis. I'm taking about a technique based on fuzzy sets. It requires a completely different methodology than all the other methods used for the same purpose.

The method is based on inductive premises like "if GDP per capita is high, then the underground economy is large" or "if taxation is excessive, tax evasion is high". To confer a membership degree to such as subjective variables depends largely on user experience.

This approach uses some concepts taken from MIMIC model. I refer to causal variables processed, namely the share of direct taxation in GDP (DIR), the share of indirect taxation in GDP (INDIR), GDP per capita Index in USD (PIBL), unemployment rate (SOM) and the corruption index (IPC). The analysed period is 1990 to 2007. The results of fuzzy approach are represented by underground economy size for 1992 – 2007 periods. Data corresponding to 1991 and 1992 are use to built arithmetic and harmonic means.

Choosing the variables listed above are not scientific motivated, but it is influenced by earlier models. I have assumed there are an interdependence relationship between these variables and underground economy. I consider that "if taxation is high and unemployment rate is high and corruption index is high and GDP per capita is low then underground economy is certainly large". This is only one of model specification, but we could create a lot of different versions:

- "If taxation is high and unemployment rate has a medium level and corruption index is slow then underground economy is medium size";

[1] First paper in economic fuzzy logic was published by Lindstrom T., A fuzzy Design of the Willingness to Invest in Sweden, Journal of Economic Behaviour and Organization, 36, 1-17, 1998.
[2] Draeseke Robert, Giles David E.A. – A Fuzzy Logic Approach to Modelling the Underground Economy, WP, Department of Economics, University of Victoria Research Grant #38163-28200, Canada, 2000, pag. 2-3.

- "If taxation is medium and unemployment rate is slow and GDP per capita is high and corruption index is low then underground economy is certainly insignificant";
- "If taxation is excessive and unemployment rate is high and corruption index is excessive and GDP per capita is medium then underground economy is large".

The choice of causal variables is subjective, but also subjective is fuzzy sets limits specification. The border between a "high taxation" and an "excessive taxation" is determined by personal choice.

Given the explanations above, I defined fuzzy sets for each of mentioned causal variables. Then I associated subjective levels with subjective values. The next step was to formulate decision rules and to establish the subjective level of underground economy using fuzzy operators. These steps were followed for each year of the data series.

Using moving average process I created a scale and I identified the subjective levels "excessive", "high", "normal", "low", "very low". The final level of underground economy for period 1992 – 1999 was subsequently adjusted because I did not take account of possible cycles that could occur in numerical data series.

The average value of the prior period was considered a "normal" value for each series. For example, the "normal" related to a variable for 2000 is the average value of the period from 1990 to 1999.

Levels around normal value were obtained as follows:

- „high": normal value + standard deviation ;
- „excessive": normal value + 2 x standard deviation ;
- „low": normal value - standard deviation ;
- „very low": normal value - 2 x standard deviation.

In other words, the algorithm is generally as:

Very low (FS)	Low (S)	Normal (N)	High (M)	Excessive (FM)
Average value – 2*ST.DEV.	Average value – ST.DEV.	Average value	Average value + ST.DEV.	Average value + 2*ST.DEV.

Table 1 represents levels corresponding to INDIR variable (the share of indirect taxation in GDP). The calculation of levels for other variables is similar. The coloured cells means possible membership levels for variable INDIR.

In 1992, the real value of this variable is 24.99%. This value must be properly framed on the membership levels:

Very low (FS)	Low (S)	Normal (N)	High (M)	Excessive (FM)
15.6249	19.7680	23.9111	28.0542	32.1973

Table 1

Fixing subjective levels for variable INDIR

	Very low (FS)	**Low (S)**	**Normal (N)**	**High (M)**	**Excessive (FM)**	**Original series (%GDP)**
1990						20.98
1991						26.84
1992	15.6249	19.7680	23.9111	28.0542	32.1973	24.99
1993	18.2806	21.2759	24.2712	27.2664	30.2617	28.98
1994	18.6592	22.0537	25.4481	28.8426	32.2371	18.10
1995	15.1575	19.5677	23.9779	28.3881	32.7983	11.47
1996	8.9843	15.4384	21.8924	28.3465	34.8006	11.99
1997	6.5176	13.4977	20.4778	27.4578	34.4379	19.09
1998	7.3425	13.8234	20.3043	26.7852	33.2661	15.77
1999	7.3045	13.5525	19.8004	26.0484	32.2963	15.82
2000	7.3554	13.3789	19.4025	25.4260	31.4495	14.88
2001	7.2405	13.1157	18.9909	24.8661	30.7413	11.23
2002	6.2785	12.3114	18.3443	24.3773	30.4102	10.16
2003	5.3022	11.5084	17.7147	23.9209	30.1272	11.62
2004	4.9160	11.0975	17.2791	23.4606	29.6421	10.58
2005	4.4276	10.6301	16.8326	23.0352	29.2377	9.92
2006	3.9286	10.1647	16.4009	22.6370	28.8731	9.41
2007	3.4464	9.7180	15.9897	22.2613	28.5329	6.49

Source: own calculation.

Anyone could note that the actual real value (corresponding to 1992) can be placed somewhere between "normal" and "high". What can we do? One of fuzzy logic advantage is that not required choosing a single level, but provides a choice of two or more. I established confidence rates, another subjective element. These membership degrees or confidence rates were built using harmonic mean.

The assigned values for variable INDIR confidence rates are:

Very low	Low	Normal	High	Excessive
(FS)	(S)	(N)	(M)	(FM)
0.0675	0.1352	0.1965	0.2551	0.3122

The associated values sum equal to 1. Observations values exceeding the upper or lower limit are treated as extreme limits. Perfect membership (complete membership) is reflected by value 1, and 0 indicates lack of belonging.

The next step is to formulate the production rules. They will determine the individual membership level for each variable. By combining these levels it will be generated underground economy size. The production rules are arbitrary.

Table 2 offer some possible production rules corresponding to causal variables for 1992:

Table 2

Possible production rules (1992)

Rule number	Causal variables					ECINF	Membership degree (Confidence rate)
	DIR	INDIR	PIBL	SOM	IPC		
1	FM	N	FS	N	M	R	0.8
2	FM	N	FS	N	FM	ME	1
3	FM	N	FS	M	FM	R	0.8
4	FM	N	FS	M	M	ME	0.8
5	FM	N	S	N	M	ME	0.8
6	FM	N	S	N	FM	R	0.8
7	FM	N	S	M	FM	R	1
8	FM	N	S	M	M	ME	0.8
9	FM	M	FS	N	M	ME	1
10	FM	M	FS	N	FM	ME	0.8
11	FM	M	FS	M	FM	R	0.8
12	FM	M	FS	M	M	R	0.8
13	FM	M	S	N	M	ME	0.8
14	FM	M	S	N	FM	R	1
15	FM	M	S	M	FM	R	1
16	FM	M	S	M	M	ME	1

Source: own calculation

Note: DIR - the share of direct taxation in GDP, INDIR - the share of indirect taxation in GDP, PIBL - GDP per capita Index in USD, SOM - unemployment rate, IPC - the corruption index;

FS – very low, S – low, N – normal, M – high, FM – excessive, E – extremely high, R – very high, ME – medium.

The rules interpretation was made by using "if-then" criterion. According to rule 1, "if the share of direct taxation in GDP is excessive and the share of indirect taxation in GDP is normal (general taxation rate is high) and GDP per capita index is very low and unemployment rate is normal and the corruption index is high then the underground economy is very high". The other rules were interpreted in the same manner. There are similar rules for each year analyzed, not exposed in these pages because the analysis is too wide. I think this fact does not affect the understanding and the relevance of the proposed approach results.

The last column of Table 2 represents the confidence rates associated to underground economy, according to the intensity of the related rule. The personal intuition and trial are involved. For example, rule 1 suggests that underground economy is not perfectly associated with level "very high". It suits this level in a proportion of 80 percents.

The last phase was to write the numerical series related to variable ECINF (underground economy). For this reason I assigned the following values (subjective values) to membership levels referred to underground economy:

- extremely high: E = 0.5;
- very high: R = 0.35;
- medium: ME = 0.20;
- low: S = 0.10;
- very low: FS = 0.05.

I mentioned again that these values choice was arbitrary. Assigning other values to membership levels, the related underground economy values will be changed.

Table 3 presents some processed information obtained by using fuzzy operators MIN and MAX:

Table 3

Setting underground economy size (ECINF)

Rule number	Causal variables					Minimum causal variables value *	ECINF
	DIR	INDIR	PIBL	SOM	IPC		
1	FM	N	FS	N	M	0.026986	R
2	FM	N	FS	N	FM	0.038350	ME
3	FM	N	FS	M	FM	0.030680	R
4	FM	N	FS	M	M	0.026986	ME
5	FM	N	S	N	M	0.026986	ME
6	FM	N	S	N	FM	0.030680	R
7	FM	N	S	M	FM	0.038350	R

Rule number	Causal variables					Minimum causal variables value *	ECINF
	DIR	INDIR	PIBL	SOM	IPC		
8	FM	N	S	M	M	0.026986	ME
9	FM	M	FS	N	M	0.033732	ME
10	FM	M	FS	N	FM	0.030680	ME
11	FM	M	FS	M	FM	0.030680	R
12	FM	M	FS	M	M	0.026986	R
13	FM	M	S	N	M	0.026986	ME
14	FM	M	S	N	FM	0.038350	R
15	FM	M	S	M	FM	0.038350	R
16	FM	M	S	M	M	0.033732	ME

Source: own calculation

* represents MIN function values multiplied with confidence rate.

4. Estimating Romanian underground economy

I calculated the maximum value recorded by variable ECINF (for the year 1992) on each membership level, as follows:

- Level R (very high): max (0.026986, 0.030680, 0.030680, 0.038350, 0.030680, 0.026986, 0.026986, 0.038350) = 0.038350;
- Level ME (medium): max (0.038350, 0.026986, 0.026986, 0.026986, 0.033732, 0.030680, 0.026986, 0.033732) = 0.038350.

These maximal values were multiplied with confidence rates values corresponding to underground economy.

Level	Maximal value	Membership level value
R	0.038350	0.35
ME	0.038350	0.20

Finally, I determined the size of underground economy corresponding to year 1992 (as a share of GDP):

$$\frac{0.038350 \times 0.35 + 0.038350 \times 0.20}{0.038350 + 0.038350} = 0.275$$

In 1992 romanian underground economy represented 27.5% from official GDP.

This working procedure has been applied to all years of the analyzed period. Summary results are presented in Figure 1.

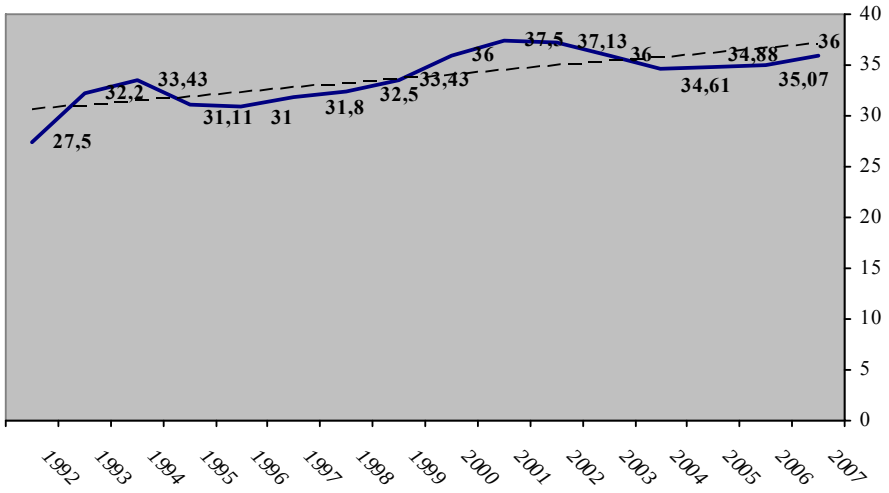

Figure 1 – *Romanian underground economy evolution using a fuzzy model*
Source: own calculations

33.76% is the average value of underground economy in Romania in the period review.

5. Conclusions

No one can sustain that a method or a model is better than other. No one can determine which methods provide the best solution, the best value or the highest confidence degree for underground economy size estimation. Why not? The nature of underground economy stays uncertain itself. However, I believe that fuzzy technique presented here may be an acceptable alternative to regression equations analysis.

Due to its clandestine or hidden nature, underground economy measurement is an uncertain and difficult matter. The fragility of methods, lack of measuring instruments and also the inherent of phenomenon nature make it difficult to use both quantitative and qualitative analysis, giving only approximations and uncertain figures. The question is if these values may be consider being normal for our country economy or exceeding the limits and endanger very seriously our national economy. The answer is given by international comparisons with similar economic structure countries.

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